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A thesis submitted to the Department of Philosophy and Psychology of the Graduate College in the State University of Iowa, in partial fulfillment of the Requirement for the degree of Doctor of Philosophy.

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HISTORICAL STATEMENT

The object in this brief historical survey is to place before the reader only those results which are most closely related to the present problem with respect to the methods used and the aspects investigated. Therefore, investigations made with similar methods but having only a narrow field, as well as those exploring a large range by different means, will not be discussed.

Preyer may be considered the pioneer worker on the problem of pitch discrimination within the tonal range. Earlies investigators concerned themselves with the least perceptible difference at only a single point in the register. Delazenne, (1) in 1827, used a metal string 1147 mm. in length with a vibration rate of 60 v.d., which he divided into equal parts by a bridge. He found that if the bridge was removed only 1 mm. from the central position a difference of pitch could be detected by well trained ears when the two parts of the string were sounded in succession. Weber (20) declared that he was able to determine the pitch of tones so accurately through the ear that he never made an error of more than one vibration on a tone of 200 v.d. He thought it possible that this keen discrimination

might be due to beats. Sauveur (9) with two monochords tuned to the same pitch found that when one string was shortened by 1/2000 of its length a difference in the pitch of the two tones was recognizable, but he leaves no record as to the pitch of the tones. Schleiber (12) recorded a differential threshold of less than .5 v.d. on the tone of b. Seebeck (14) and two superior violin players differentiated without fail between two tuning forks vibrating at 1209 and 1210 v.d.

Preyer.—Preyer (7) used the Appunn tonmesser,—an instrument consisting of reeds which gave the following tones: from 500 to 501 in steps differing by 0.1 v.d., 504, 508, 512, 1000 to 1001 in steps of 0.2 v.d., 1008, 1016, 1024, 2048, and 4096 v.d., respectively. More than a thousand judgments as to whether the two tones compared were equal or different, were secured from twelve practiced observers. They were not required to tell the direction of the difference. To his own results he added those of Delezenne and Seebeck. Transcribing Delezenne's measurement in millimeters into terms of frequency of vibration, he found that the shorter string produced a tone of 120.2 v.d., while the longer one gave a tone of 119.8 v.d., thus making a difference of 0.4 v.d. which skilled observers could detect. Likewise, a simple computation revealed the fact that Seebeck had the very low threshold of 0.36 at 449 v.d. After some verification of Seebeck's results, Preyer inclined to the belief that the threshold might be brought as low as 0.25 v.d. The combined results give differential thresholds for four different places in the tonal range as follows: Delezenne at 120 v.d. found the threshold to be 0.4 v.d., Seebeck at 440 v.d. found it to be 0.4 v.d., Preyer at 500 v.d., 0.3 v.d., and at 1000 v.d., 0.2 v.d. From this study, inadequate as it may seem, Preyer draws the following conclusions:

(1) One-third of a vibration on 500 v.d. and five-tenths of a vibration on 1000 v.d. will always be recognized as different by the best observers, although the most sensitive, the most trained, and the most reliable ears tested could not recognize a difference of 1000 and 1000.25 v.d. nor of 500 and 500.2 v.d. (2) Very high tones and very low tones cannot be discriminated so accurately as tones of the middle region. Capacity for discrimination is keen between c and c^3, keenest for the region from a^1 to c^2, but beyond c^3 it decreases slowly until it becomes very unreliable at c^5. Fis^4 marks a second point of keenness of capacity. (3) The relative difference for pitch is dependent, in a high degree, on the number of vibrations

of the compared tones; and the absolute difference-sensitiveness does not decrease with the pitch. (4) The judgment concerning the place of tones in the tonal line is more uncertain than the judgment as to whether the two tones lie at different points. (5) Practice is an influential factor in pitch-discrimination. Extreme fineness of capacity is peculiar only to those who have much familiarity with tones.

Luft.—In the Leipsic laboratory, during the years 1884 to 1886, Luft (4) experimented upon a range of tones extending from 64 to 2048 v.d. Tuning forks were used for the production of the tones, the variable being mistuned from the standard by means of sliding weights. The forks 64, 128, and 256 v.d. were energized by the stroke of a hammer of India-rubber, forks of 512 v.d., by means of a violin-bow, and forks of 1024 and 2048 v.d., by a wooden hammer, which was padded with felt. Forks of 64 v.d. were held upon large resonator-boxes; 128, 256, and 512 v.d. were brought to the openings of resonator-tubes of paper; while 1024 and 2048 v.d. were permanently attached to resonator-boxes. He used the method of minimal change and employed from four to eight different steps in passing from a large difference to one that was just perceptible, with the following results:

Standard v.d.	64	128	256	512	1024	2048
Difference	.15	.16	.23	.25	.22	.36

"In the field of tonal quality within the region investigated the psychophysical law, according to which the absolute differences of sensation correspond to relative differences of stimulus which must be constant, finds no application. On the contrary the differential threshold within the interval mentioned approaches the constant average of 0.2 vibrations."

The threshold value slowly rises from 64 to 2048 v.d. with the single exception of 1024 v.d. Luft admits that the error here is probably due to an objective circumstance. His results can scarcely be compared with those of Preyer as the methods employed were quite different. It is a point worthy of observation, however, that with Preyer the point of finest discrimination lies at 500 v.d. while with Luft it lies at 64 v.d. Later, Luft found the threshold of 32 v.d. (by the same method employed with 64 v.d.) to be about .44 v.d. His results, therefore, do not contradict Preyer's statement that very high tones and very low tones cannot be distinguished as readily as tones of the middle register.

Luft noted that practice lowers the threshold and that the effect

of practice is not equally distributed in all parts of the range. The influence of practice is of special importance at 64 and 128 v.d. Luft lowered his own record from 0.85 to 0.3 v.d. at 128 v.d., and from 0.42 to 0.15 v.d. at 64 v.d. He believes that the only reason that the initial thresholds for the lower tones is higher than those of the central region, is, that the degree of practice for the former is not so great. The variable of practice was far less noticeable in the higher regions which was due, he believed, to the greater intensity and persistence of these tones. He even ventured to say that the individual variations of the differential threshold are, for the most part, due to practice.

Meyer.—In 1898, Meyer (5) published Professor Stumpf's thresholds for the discrimination of pitch. Tuning forks were used for the tones 100, 200, 400, 600, and 1200 v.d. The variable forks were mistuned by the insertion of a screw in the end of the prong,—a more accurate device than that of sliding weights. After discarding the method of minimal change as practically worthless for the problem in hand, and thereby questioning the validity of Luft's results, Meyer adopted the method of right and wrong cases. Forks of 100 and 200 v.d. were held in the hand and brought to the openings of the resonators, while forks of 400, 600, and 1200 v.d. were mounted on resonator-boxes and were energized by the blow of a hammer. Each individual experiment was performed three times and even more if the observer wished it, before a judgment was required. In this manner Meyer thought to equalize variations of intensity and time-interval, as well as fluctuations of attention. Stumpf's thresholds as determined by means of the Cattell-Fullerton table, from the data given, are as follows:

V. D.	100	200	400	600	1200
Differential threshold	.54	.25	.28	.24	.69

The author concluded his report thus:

"One sees, therefrom, that approximately the same difference of pitch is recognized with equal certainty at 200, 400, and 600 v.d. and with less, but likewise moderately equal certainty, at 100 and 1200 v.d. The differences in these cases are so small that they may be considered accidental. That the certainty of judgments declines in still higher and still lower tones is self-evident."

Stücker.—Stücker's work (16) is the most extensive study published on this particular subject. His observations covered the range between the limits 72 and 35000 v.d., or nine entire octaves. He

employed the following standard tones: d^{-1} (73.4), c^0 (130.5), c^1 (261), all the tones of the major scale up to c^2 (522), a^2 (870), a^3, g^3 (3100), c^5, g^5, c^6, g^6, c^7 and c^8. All of the tones up to and including c^2 were produced with tuning forks, a^2 and a^3 with a monochord, and the remaining ones with a Galton whistle. In each individual series he started with a large difference in the number of vibrations of the two instruments and then made the difference gradually smaller until the threshold was reached. Such a procedure was repeated a few times for the purpose of verification. Whether the observer indicated the direction of the difference or merely the difference is not stated. Given below are the average values of the relative and absolute sensitiveness of discrimination of his fifty observers for eight different levels, with his statement in summary:

Pitch	d^1	c^0	c^1	a^1	a^2	a^3	g^3	g^5
Rel. Disc.	.94	.74	.49	.32	.30	.44	.86	4.91
Abs. "	.7	1.	1.3	1.4	2.5	7.7	26.7	304

(1) Neither the absolute nor the relative sensitiveness to difference of the two tones remains constant in the different tonal regions. (2) The relative difference-sensitiveness is in general the greatest in the first and second accented octaves; in many cases, however, the second maximum lies in the third and fourth accented octaves. (3) With one-third of the entire number of observers the relative sensitiveness to difference in the second half of the first accented octave is nearly equal; namely, 0.2 and 0.3; when one compares the individual curves of sensitiveness with these, the places of greatest sensitiveness lie in the upper half of this region, while with unmusical individuals they occur in general in the lower half. (4) The degree of sensitiveness is subjected to fluctuations within an octave, which is repeated in each octave in the same proportion; it is the greatest for c, slightly less great for g and still less for f and h. (5) A number of persons possess a secondary maximum of sensitivity. (6) An unusually great sensitiveness in high tonal regions is a characteristic of musical persons.

Stücker points out that the discrimination was far more accurate in the lower regions when the second tone was lower, while in the higher region the opposite was true. The inference here is, that judgments are facilitated when the second tone is farther removed from the first and second accented octaves, which are most frequently employed in musical composition; i.e., when the second tone is the farther from this middle register, the judgment seems to be more accurate. He further adds, that the daily variation of non-musical observers is less than for musical ones.

A year later this same author (17) published a report supplementing the one just reviewed. In this he states the results obtained from three different types of observers, professional players of various instruments, singers, and individuals decidedly unmusical. The average values of the absolute differences for the three different sets of observers have been computed for seven levels in the tonal range, as follows:

	d^1	c^0	c^1	a^1	a^2	a^3	g^4
Players	.35	.37	.40	.56	1.20	2.64	13.0
Singers	.46	.48	.44	.71	1.62	3.07	14.0
Unmusical	12.62	2.20	2.80	4.80	9.96	24.00	130.0

Of special interest in this second article is the statement that with tenors and sopranos the finest discrimination is found beneath their voice register, but with bass and alto singers above their voice register; the difference is not between the voices of men and women, but only appears between the relative height and depth of the voice-register of both sexes.

The age difference, he maintains, is more significant than that between musical and non-musical observers. After the age of thirty, sensitiveness to difference declines and the range becomes restricted.

Schaefer.—In 1910, Schaefer (10) submitted a thesis to the Department of Psychology of the State University of Iowa on the subject, "The Curve for the Variation of Pitch Discrimination within the Tonal Range", which has not been published. The apparatus and method were practically the same as those used in the present investigation. For observers, he had fifteen normal individuals varying in musical ability and training. Five hundred tests were given on each of the tones 24, 32, 64, 128, 256, 512, and 2048 v.d. The average threshold for each of these in the order given is as follows: 3.3, 3.4, 2.9, 1.3, 1.5, 1.8, and 6.7 v.d. He summarizes his results thus:

> (1) The form of the composite curve indicates that discrimination for the average normal individual is most difficult in the higher and the lower registers and becomes easier in the middle register. (2) The majority of the individual curves are of the same form as the composite. Curves of individuals having high thresholds are of about the same form as the curves of individuals having low thresholds. (3) There are notable individual differences. (4) Musical training does not influence to any large extent, the ability to perceive difference of pitch. (5) It is easier to detect difference in pitch than to name the direction of the difference.

STATEMENT OF THE PROBLEM

The primary purpose of this investigation has been to determine the prevalence of islands or gaps in pitch-discrimination within the tonal range. The pursuit of this aim has taken the form of an attempt to make a comparatively large number of complete individual measurements on pitch-discrimination within the tonal range with as many as possible of the hitherto unknown or disregarded sources of error under control. On the basis of frequently observed defects in the hearing of pitch, found in clinical cases, it is generally believed that such disturbances occur in varying degrees in normal persons. In the curves of two or three of Schaefer's observers, there are places where discrimination of pitch is less keen than the balance of the curves would seem to indicate that it ought to be; in the case of one observer the evidence of a gap was striking. Professor Titchener (19) deems such cases of sufficient importance to bring to the support of the Helmholtz theory of hearing. He says:

"Cases occur in which the range of hearing is normal, but the tonal scale is not continuous; there are tonal gaps, large or small, parts of the scale where the patient is completely deaf to tonal stimuli, though he can perfectly well hear the cases above and below."

The sources of error in a problem of pitch-discrimination are so great and insistent that successive investigators of the same problem are fully justified in a patient struggle to overcome them with progressive insight. In reading the various reports on the subject, one cannot help being impressed with the fact that very few, if any, of the investigators fully realized the significance of the many important variables which could easily—and doubtless did—vitiate the results. The disturbing factors, due to faulty apparatus and inadequate procedure, mentioned by Professor Seashore in his preliminary report (13), suggest the seriousness of the problem. From my own experience I am convinced that his statement in regard to these factors is in no way exaggerated. Rather, it has not been made sufficiently emphatic. The danger of false criteria entering into the judgments of the most conscientious observer, either consciously or unconsciously, can scarcely be realized by one who has not encountered them first-hand. The danger of identification, alone, is sufficient to make the investigator very cautious.

Apparatus and Method.—In this investigation the measurements

were made at six different levels in the register; namely, 64, 128, 256, 1024, and 2048 v.d. The tones were produced by the best grade of Kohl tuning forks. For 128, 256, 512, and 1024 v.d. Helmholtz resonators were used; the forks of 2048 v.d. were mounted on resonator-boxes; while resonance for 64 v.d. was produced by extending the Helmholtz resonator for 128 v.d. For 64 v.d. a second set of forks was found to be more satisfactory at a later stage of the experiment. These were made of round tool steel, 12 mm. in diameter. The prongs were 30 cm. in length and carried hard rubber discs 10 cm. in diameter.

The sounder was a simple device consisting merely of a lead pipe about one inch in diameter with one end bent into the form of a circle for the base, and the other in the shape of a U at right angles to the base. The U-end, when covered with several thicknesses of rubber, made a sounder of the required elasticity and softness. The placing of the sounder on leather sand-bags resting on a heavy metal stand eliminated, in large part, the accessory noise of the blow. The forks of the four central octaves were energized by striking the middle of the prong upon the sounder; the forks of 2048 v.d. were struck as lightly as possible with a felt-hammer; while those of 64 v.d. were set into vibration by striking them on the sand-bags.

To mistune the variable fork, in every case except those of the lower limit, a screw was inserted in the end of each prong and to these were attached nuts, varying in weight, to give the desired pitch. Such a device is a decided improvement over the method of sliding weights, inasmuch as the latter may allow a slight change in position, with a corresponding change in pitch, during the course of the experiment. This is especially true of the smaller forks. At 64 v.d. variation in pitch was secured by shifting the discs, which were firmly attached to the forks by large set screws. At each of the steps the successive differences of one, two, three, five, and eight vibrations were chosen—a range which was found to be sufficient for all but one or two observers. All of the forks were tuned to an accuracy of five-hundredths of a vibration per second.

The mode of procedure followed the plan suggested by Professor Seashore in his preliminary report (13) in almost every respect, in the four central octaves where it was possible to do so. A most careful attempt was made to keep the tones at a constant intensity

without resorting to the uniformity of mechanical devices. The experimenter simply relied on the accuracy of his own hand and ear in presenting the forks in such a way that the tones would be of equal strength. If at any time, through a lapse on the part of the experimenter, the difference of intensity seemed pronounced, the trial was repeated. Mechanical devices are particularly unsatisfactory in that the difference of intensity which is practically certain to occur, be it ever so slight, is constant and might thus become a criterion for identification. In the method of presentation by hand, this source of error is eliminated. The ideal presentation is that in which the tones are just loud enough to be heard without a strain of the attention, and extreme care was taken throughout to guage the tones by this standard. The duration of each tone, as well as the time-interval, was approximately one second. Whether the constant or the variable tone should be presented first, was decided by a key which had been arranged first by chance and then revised to the extent that the same order could be followed no more than three times in succession, and that in one hundred tests the two possible sequences should have the same frequency. The observers in every case were required to render their judgments in terms of "second tone lower", or "second tone higher", in accordance with the method of right and wrong cases. No doubtful judgments were allowed; when the observer felt uncertain after repeated tests he was simply requested to guess. As a rule each individual experiment was given but once, but whenever disturbances of any sort, either objective or subjective, were noted, the experiment was repeated. Observers were instructed to trust the first impression. Except with the lowest tones, where the judgments were given orally and were then recorded by the experimenter, the observers themselves kept the record by simply writing *H* or *L* as an abbreviation of the judgments "higher" or "lower." With one observer, however, the response was oral throughout because attention to the writing caused too much of a distraction. At least one hundred judgments were recorded at each level, but many observers required a considerably larger number before their thresholds could be satisfactorily determined. No series of observations extended long enough to cause any disturbing fatigue. The monotony of the experiment was broken at intervals by the checking of the record and by the adjusting of the forks. Fatigue caused previous to the experiment could

not be very well controlled as the observers had to be taken at times which best suited their convenience. The tests were, however, fairly well distributed throughout the hours of the day and those observers who did come at a late hour were always dismissed if they felt fatigue to a degree which they thought might interfere with their best work. The experiment was conducted in the sound-proof room and in every instance the observers were tested individually so that distractions of an objective character were reduced to conditions connected only with the actual experiment.

The experimental control was naturally most difficult at 64 v.d. The large size of the forks not only made them more difficult to handle but also increased the possibility of overtones. Still another problem was presented in obtaining sufficient resonance for these tones of low intensity. Overtones were especially distracting with the first pair of forks that were used, but it was possible to overcome them to some extent by setting the forks in heavy handles of iron and by putting heavy rubber bands upon the prongs. Yet the increased weight added to the difficulty of handling. Two different methods were tried with these forks; namely, bringing the forks to the openings of the resonators, described above, and presenting them to the ear of the observer without the aid of a resonator. Both of these methods are unsatisfactory. The resonator scarcely makes the tones loud enough to make the judgment one of certainty, and it is difficult for the experimenter to present the tones so that they are of equal intensity. Holding the forks to the ear has the advantage of making the tones louder, but here again the variable of intensity is left uncontrolled, and the possibility is open to the observer for obtaining clues from the position of the fork, from timbre, and from noises caused by movements in presentation. The fact that the tones could be distinctly heard by the second method gave it the preference. But when the results were compared with those obtained for 128 v.d. the thresholds seemed abnormally large. This pair of forks was therefore discarded for the forks with the discs, which were found to answer the purpose much better, for at least three reasons; namely, they were freer from overtones, the tones were louder and clearer because of the increased vibratory surface offered by the discs, and they were neither so heavy nor so long, which facilitated handling very materially. With these forks the method of presentation to the ear was adopted, but on

account of the louder tone it was possible to hold the forks farther from the ear. Being also lighter in weight, they could be energized in a more uniform manner, and it was easier to bring them more nearly to the same point opposite the ear; thus the variable of intensity and direction of source could be more adequately controlled. An opportunity was not offered for the retesting of all individuals whose thresholds had been determined by the first pair of forks, but in most cases where a second was possible, somewhat lower thresholds were obtained.

No particular comment in regard to the forks of 128 and 256 v.d. is necessary. They were energized and presented to the resonators with the conditions of duration, time-interval, and intensity carefully controlled. In each case the tones were perfectly clear and distinct. The forks producing these tones held up long enough to allow five individual experiments without restriking. But the control was not quite so satisfactory at 1024 v.d. The forks at this level would vibrate with sufficient energy for only two tests. A more forceful blow was also required, and it was necessary to bring them very close to the small resonators, indeed so close that they nearly touched it. All this, of course, made it more difficult for the experimenter to maintain a constant intensity. Again, the piercing character of the tone was annoying to some observers. The tones produced by forks of 128, 256, and 512 v.d. were not heard by the observers except when reinforced by the resonators. But the 1024 v.d. forks gave a high piercing tone before being presented to the resonator. The observer, as much as possible, ignored this tone and concentrated his attention on the tones as they were intensified by the resonators.

In the upper limit, the method was necessarily quite different. The small resonator-boxes on which the forks were mounted were held in the hand; the one fork was struck and dampened, and then the second in close succession. So delicate a stroke was necessary to produce a tone that the noise of the blow was but a slight distraction, if any. It was extremely difficult, however, to keep the intensity constant. To eliminate discrimination of the direction of source, the position of the left hand was shifted to bring the forks to exactly the same place before they were energized.

Of the fifty observers who made this study possible by giving it their time and thought, thirty-three were members of the elemen-

TABLE I. Absolute differential thresholds

Obs.	64 v.d. T	64 v.d. m.v.	128 v.d. T	128 v.d. m.v.	256 v.d. T	256 v.d. m.v.	512 v.d. T	512 v.d. m.v.	1024 v.d. T	1024 v.d. m.v.	2048 v.d. T	2048 v.d. m.v.
1	2.5	0.9	1.9	0.5	0.8	0.6	0.7	1.1	3.3	.0	5.3	0.4
2	3.5	0.1	0.8	0.6	0.6	0.8	1.0	0.8	2.2	1.1	2.5	3.2
3	3.3	0.1	0.6	0.8	0.7	0.7	0.8	1.0	2.5	.8	5.6	0.1
4	3.0	0.4	1.5	0.1	2.2	0.8	2.3	0.5	1.1	2.2	3.5	2.2
5	4.0	0.6	1.3	0.1	1.1	0.3	0.9	0.9	4.1	.8	7.3	1.6
6			0.7	0.7	0.8	0.6	1.2	0.6	2.4	.9		
7	4.0	0.6	0.7	0.7	1.2	0.2	1.7	0.1	3.5	.2	6.7	1.0
8	1.0	2.4	1.5	0.1	1.4	0.0	1.8	0.0	3.7	.4	6.5	0.8
9	6.4	3.0	2.7	1.3	0.7	0.7	1.7	0.1	5.3	2.0	6.5	0.8
10	1.5	1.9	1.4	0.0	1.1	0.3	1.8	0.0	2.4	.9	4.9	0.8
11	2.0	1.4	0.8	0.6	1.5	0.1	1.0	0.8	3.9	.6	5.6	0.1
12	3.7	0.3	0.7	0.7	1.5	0.1	2.4	0.6	4.3	1.0	3.5	2.2
13	1.5	1.9	1.0	0.4	0.7	0.7	2.0	0.2	3.8	.5	6.7	1.0
14	4.0.	0.6	2.7	1.3	1.8	0.4	2.2	0.4	5.0	1.7	10.0	4.3
15	3.0	0.4	1.4	0.0	2.1	0.7	4.4	2.6	6.4	3.1	8.8	3.1
16	3.4	0.0	1.3	0.1	0.6	0.8	0.8	1.0	2.2	1.1	3.0	2.7
17	2.0	1.4	1.1	0.3	2.9	1.5	5.0	3.2	5.1	1.8	7.6	1.9
18	0.7	2.7	1.0	0.4	2.0	0.6	0.6	1.2	0.8	2.5	5.8	0.1
19	5.2	1.8	0.6	0.8	1.1	0.3	0.8	1.0	4.1	0.8	9.7	4.0
20	1.0	2.4	1.0	0.4	0.7	0.7	1.2	0.6	3.2	.1	6.1	0.4
21	3.8	0.4	2.3	0.9	1.6	0.2	1.7	0.1	3.6	.3	5.5	0.2
22	2.5	0.9	0.8	0.6	1.0	0.4	1.3	0.5	2.0	1.3	3.0	2.7
23			2.0	0.6	1.3	0.1	1.4	0.4	6.8	3.5	5.5	0.2
24	6.4	3.0	1.5	0.1	1.1	0.3	1.5	0.3	2.0	1.3		
25	4.0	0.6	2.0	0.6	2.5	1.1	2.5	0.7	2.4	.9	4.4	1.3
26	3.0	0.4	0.7	0.7	1.0	0.4	1.9	0.1	1.8	1.5	5.8	0.1
27	4.7	1.3	0.7	0.7	1.5	0.1	1.7	0.1	2.1	1.2	3.6	2.1
28	3.4	0.0	0.6	0.8	0.8	0.6	1.1	0.7	2.3	1.0	3.2	2.5
29	2.4	1.0	2.1	0.7	1.5	0.1	1.6	0.2	8.4	5.1	10.2	4.5
30	2.5	0.9	2.4	1.0	1.3	0.1	1.4	0.4	4.1	.8	5.7	0.0
31	6.4	3.0	2.4	1.0	2.0	0.6	1.5	0.3	3.9	.6	3.0	2.7
32	2.4	1.0	1.0	0.4	1.0	0.4	1.1	0.7	2.2	1.1	4.6	1.1
33			1.7	0.3	1.4	0.0	2.7	0.9	3.2	.1	8.8	3.1
34	3.3	0.1	1.5	0.1	1.5	0.1	1.4	0.4	2.5	0.8	4.9	0.8
35	6.4	3.0	1.1	0.3	1.7	0.3	1.0	0.8	6.4	3.1	9.9	4.2
36	8.8	5.4	1.1	0.3	0.8	0.6	2.2	0.4	3.5	.2	4.8	0.9
37	3.0	0.4	1.5	0.1	1.4	0.0	2.6	0.8	4.4	1.1	10.2	4.5
38	0.7	2.7	0.6	0.8	0.7	0.7	2.1	0.3	1.6	1.7	3.7	2.0
39	7.2	3.8	4.1	2.7	3.2	1.8	1.6	0.2	7.6	4.3		
40			0.9	0.5	1.2	0.2	0.9	0.9	0.7	2.6	4.3	1.4
41	2.4	1.0	0.8	0.6	0.7	0.7	2.1	0.3	1.8	1.5	3.5	2.2
42	4.0	0.6	2.0	0.6	3.2	1.8	4.9	3.1		7.0	7.0	1.3
43	1.3	2.1	0.8	0.6	0.7	0.7	1.1	0.7	1.8	1.5	5.3	0.4
44	3.0	0.4	1.1	0.3	0.4	1.0	1.2	0.6	2.6	.7	3.0	2.7
45	3.1	0.3	1.4	0.0	1.5	0.1	1.4	0.4	3.2	.1	8.0	2.3
46			1.4	0.0	1.3	0.1	2.6	0.8	2.7	.6		
47	3.0	0.4	1.2	0.2	1.6	0.2	1.7	0.1	2.4	0.9		
48	3.0	0.4	1.4	0.0	1.1	0.3	0.8	1.0	2.7	.6		
49	5.0	1.6	4.0	2.6	2.5	1.1	6.1	4.3				
50	1.3	2.1	1.0	0.4	0.7	0.7	1.0	0.8	0.9	2.4	1.2	4.5
Mean	3.4	1.5	1.4	.57	1.4	.5	1.8	.76	3.3	1.31	5.7	1.56
Median	3.00		1.2		1.3		1.5		3.0		5.5	

TABLE II. Relative differential thresholds

Obs.	64 v.d.	128 v.d.	256 v.d.	512 v.d.	1024 v.d.	2048 v.d.
1	.31	.12	.03	.01	.03	.02
2	.44	.05	.02	.02	.02	.01
3	.41	.04	.02	.02	.02	.02
4	.38	.09	.07	.04	.01	.01
5	.50	.08	.03	.01	.03	.03
6		.04	.03	.02	.02	
7	.50	.04	.04	.03	.03	.03
8	.13	.09	.04	.03	.03	.03
9	.80	.17	.02	.03	.04	.03
10	.19	.09	.03	.03	.02	.02
11	.25	.05	.05	.02	.03	.02
12	.46	.04	.05	.04	.03	.01
13	.19	.06	.02	.03	.03	.03
14	.50	.17	.06	.03	.04	.04
15	.38	.09	.07	.07	.05	.03
16	.43	.08	.02	.01	.02	.01
17	.25	.07	.09	.08	.04	.03
18	.09	.06	.06	.01	.01	.02
19	.65	.04	.03	.01	.03	.04
20	.13	.02	.02	.02	.03	.02
21	.48	.14	.05	.03	.03	.02
22	.31	.05	.03	.02	.02	.01
23		.13	.04	.02	.05	.02
24	.80	.09	.03	.02	.02	.02
25	.50	.13	.08	.04	.02	.02
26	.38	.04	.03	.03	.01	.02
27	.59	.04	.05	.03	.02	.01
28	.43	.04	.03	.02	.02	.01
29	.30	.13	.05	.03	.06	.04
30	.31	.15	.04	.02	.03	.02
31	.80	.15	.06	.02	.03	.01
32	.30	.06	.03	.02	.01	.02
33		.11	.04	.04	.03	.03
34	.41	.09	.05	.02	.02	.02
35	.80	.07	.05	.02	.05	.04
36	.11	.07	.03	.03	.03	.02
37	.38	.09	.04	.04	.03	.04
38	.09	.04	.02	.03	.02	.01
39	.90	.26	.10	.03	.06	
40		.06	.05	.01	.01	.02
41	.30	.05	.02	.03	.01	.01
42	.50	.13	.10	.08		.03
43	.16	.05	.02	.02	.01	.02
44	.38	.07	.01	.02	.02	.01
45	.39	.09	.05	.02	.03	.03
46		.09	.04	.02	.02	
47	.38	.08	.05	.03	.02	
48	.36	.09	.03	.01	.02	
49	.63	.25	.08	.10		
50	.16	.06	.02	.02	.01	.01
Average	.4	.09	.04	.03	.03	.02

tary class in psychology in the University, sixteen others were advanced students in psychology, and one other a member of the staff in psychology. It is important to note that the fifty represent a selected group. The thirty-three from the elementary class were chosen from a class of one hundred or more because their differential thresholds at 435 v.d. were less than 8 v.d., as determined from a test given to the class for purposes of demonstration. The advanced students had likewise shown in previous tests that their thresholds for discrimination of pitch were easily less than 8 v.d. Their closer association with the work in the department of psychology also tended to make them slightly better as a group than the elementary students. This basis of selection must be borne in mind in the consideration of the results, for our composite curve is not an average curve; it is superior to the average. It was gratifying to find that all of the observers took keen interest in the problem and made a sincere effort to give the work their best attention. Their knowledge of the fact that they were chosen because of their former good record helped them to maintain an interest.

RESULTS

The Composite Curves.—Table I includes the individual thresholds in terms of the absolute difference of vibrations for the six points in the range. The odd numbers of the observers refer to women, and the even, to the men. The thresholds are given in column T, and the mean variation in column m.v. At the foot of the table are the mean, the median, and the mean variation of the group. In Table II the same records are reduced to the relative threshold expressed in terms of the fractional part of a whole tone, at the respective levels. The figures in italics at the head show the number of vibrations in a whole tone at each of these respective levels. The record of Table I is shown graphically in Fig. 1 and that of Table II in Fig. 2. By an error the decimal point was left out before each of the numbers 1, 2, 3, and 4, in Fig. 2.

There is evidently no essential difference between the mean and the median curves; they run practically parallel throughout their course, coming a little closer together at 256 v.d. than at any other point. But inasmuch as the mean allows the extremes an influence out of proportion to their importance, the median must be considered the truer representative figure.

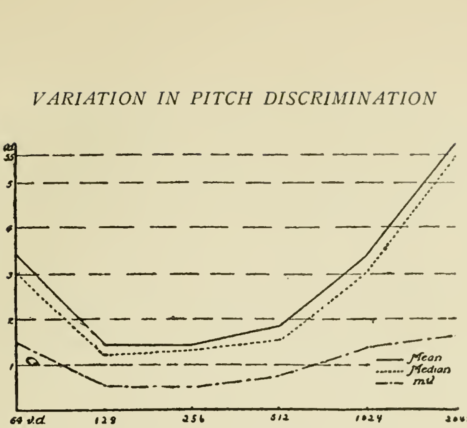

Fig. 1. Mean, median, and mean variation—absolute (Table I).

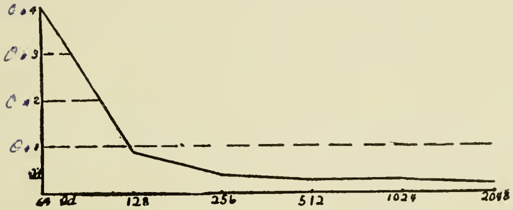

Fig. 2. Mean—relative (Table II).

The average capacity of discrimination, as measured in terms of absolute difference, is practically the same for 128 and 256 v.d. From this central register the curves rise slowly to 512 v.d., from which begins a rapid rise that becomes even more rapid from 1024 on up to 2048 v.d. The curve of mean variation (Fig. 1) follows the general trend of the composite, which means that the thresholds for this group of fifty form a more compact grouping in the central register, while at both the upper and the lower limits they are more widely separated.

The relative curve declines rapidly at first and then very gradually, reaching its lowest point at 2048 v.d. In other words discrimination of pitch, as measured in fractional parts of a whole tone, decreases somewhat abruptly from 64 to 128 v.d., but very slowly from that point to the upper limit of the range studied. In the relative curve the minimal value is at 2048 v.d. which is the region of the maximal value in the curve of absolute difference.

The absolute difference of vibration frequency has been adopted as the vehicle of expression in this report, for the reason that it is slightly more concrete and brings out the individual differences more strikingly. Its true relation to the relative must, however, be kept in mind.

The curves represent with a high degree of accuracy, it is believed, the average capacity of a group of observers such as have had a part in this study. But there is little doubt that the form of the curves has been influenced, to some extent, by certain factors other than those of actual discrimination of pitch. There are objective factors which could not be perfectly controlled and which in some cases have led to confusion, but in other cases have resulted in identification. The former necessarily raised the threshold, while the latter lowered it. The subjective variables of attention and practice are important inasmuch as attention is seldom at is best and then only for short duration, and the degree of practice might always be greater. The thresholds are therefore not quite as low as they would be under the most ideal conditions.

With tuning forks, it is impossible to produce as satisfactory a tone at the extremes as in the central register. Discrimination of pitch at 64 and at 2048 v.d. is thus made most difficult and the observer has a tendency to pick up other criteria than pitch upon which to base his judgments. Differences of intensity, change in the direction of the source of sound, and noises accompanying the control of the experiment are the chief factors which cause disturbance. They lead to confusion, rather than to identification, because the method used necessitated their approximately equal distribution between the higher and the lower tones; that is to say, that they occurred in a chance order, were therefore unpredictable, and consequently could not be used as safe criteria for accurate judgments; for example, if an observer was inclined to judge the more intense tone the higher, there would be an increased probability of error whenever the lower tone happened to be more intense. Had the forks been energized by a mechanical device, rather than by the free hand, these variables would have been constant and would have become a means of identification, rather than a source of confusion. At the higher limit it was difficult to keep the tones of equal loudness. The tones produced by the small forks are very fine and persistent, and a slight variation in the forces of the blow produced

a perceptible change in the intensity of the tones, which was often confusing. Whether or not the greater intensity favored a judgment of higher or lower varied with the individual. For some, the pitch being nearly equal, the louder tone was considered the higher, while for others the reverse experience was true.

It is in the lower limit, however, that the most abrupt rise in the threshold is to be found. As has been previously mentioned, various methods of presentation were given a trial, but none of them, excepting with a very few observers, gave results which were comparable with those obtained at 128 v.d. Only two observers had a lower threshold for 64 and for 128 v.d., (Nos. 8 and 18). For observer No. 20 the thresholds for the two tones were the same, while No's. 10, 13, 29, 30, 38, and 50 were the only remaining ones whose thresholds for 64 v.d. did not exceed that of 128 v.d. by more than 0.5 v.d. In other words, forty observers have a threshold for 64 which is more than one-half of a vibration higher than for 128 v.d. That this difference would have been less had it been possible to rule out all the factors of confusion is probable.

But not all of the variables cause confusion. Those which are constant soon come to be associated with one of the two possible judgments and this, in time, brings about a lowering of the threshold. Just what is seized upon as a means of identification one cannot always say. The auditory capacity of analysis is very keen and often the slightest variable which occurs in a particular setting is selected as a clue for the proper response. Slight variations in timbre are among the most frequent sources of identification. It is impossible to make two forks exactly alike and the unavoidable structural difference may be perceived in the nature and composition of the overtones. The forks of the lower limit are particularly susceptible to variation in timbre. If these differences are perceptible, the error of identification is sure to appear. Even with presentation by hand there is the possibility of the experimenter's falling into some characteristic habit of presenting the forks, which may be identified eventually. He may form the habit unconsciously of striking one fork at a different angle from that of the other, or the time-order may have some constant peculiarity which gives a clue.

The errors due to identification are without a doubt the most serious with which the experimenter has to contend. But in an experiment such as this the error of identification is usually discover-

able by comparing the thresholds of one level with those of the other levels. Whenever an observer has a threshold at any particular level considerably lower than the tentative norm would warrant, the chances are, that the error of identification has had a part to play. The record of No. 20 is wanting in the table for 2048 v.d. because he had discovered some criterion other than pitch upon which to base his judgments. In fact he made nearly a perfect record with a difference of one vibration,—a lower threshold than his records at the other levels would warrant. Two other observers had a similar experience in the upper limit, but when the method was slightly changed, they lost their clue and were forced to rely on pitch. In the lowest level Nos. 28 and 40 were influenced in some way by criteria other than pitch, the latter to such an extent that his results were worthless. As has been said, just what criteria were selected by these observers is not known. Their introspections fail to reveal them, the observers contending throughout—and with undoubted conviction—that they were judging on pitch alone. Such illustrations show that the experimenter connot be too careful in his attempt to keep the judgments confined to pitch.

The subjective variables of attention and practice also play more important rôles at the extremes than in the central register. To secure a low threshold at these levels closer attention is necessary and, as these tones are rarely heard, the degree of practice is much less than for tones of the central register. Practice for these tones is only to be had in the laboratory as they are seldom used in musical compositions. Observers Nos. 2, 28, 47, and 50 were the only ones who had the advantage of practice for these extreme tones and their thresholds at these levels are all below the average.

Summing up, we have found that the curve of pitch-discrimination shows the threshold of absolute difference to be keenest from 128 to 256 v.d.; from 256 to 512 v.d. it takes a gradual rise; and from 512 to 2048 v.d., a rapid rise. On the lower side, from 128 to 64 v.d., the rise is very sudden. As expressed by the curve of relative difference, there is a continual decline from the lower to the higher limit; this decline, however, is very rapid from 64 to 128 v.d., much less pronounced from 128 to 256 v.d., and from 256 to 2048 v.d. the curve becomes very nearly a straight line. It will be of interest now to compare the above results with those of other investigators.

Comparative Curves.—Figure 3 represents the composite results

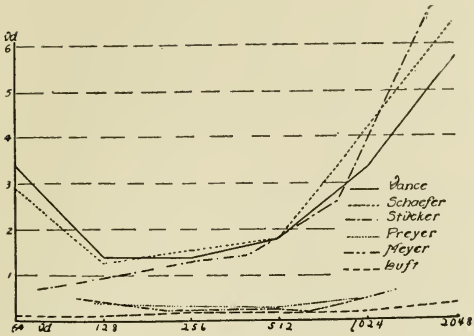

Fig. 3. Composite curves for six different investigators—absolute.

of the six different investigators who have approached the problem by the same general method. The curves of Stücker (16), Schaefer (10), and the writer show considerably higher thresholds than those of Luft, Preyer and Meyer. This difference may be explained. In a pioneer work, such as Preyer's (7), numerous sources of error as yet undiscovered must have had much influence upon the results. His low thresholds can be attributed, in some degree at least, to the error of identification. Since this error may creep in when the best grade of tuning forks is used, there is little doubt but that it must have played an important rôle with the tonmesser. Furthermore this instrument, the reed, is not reliable for fine pitch differences. Luft's (4) values, especially at the extremes, must likewise be questioned. Whenever such low results are obtained at 64 and at 2048 v.d., there must be conclusive evidence that they are not due primarily to discrimination of pitch, but to some other factor which permits of identification. Luft has given no such proof. Furthermore, in a problem of this kind, the method of minimal change, which he used is unreliable, as Meyer has well pointed out, in that it introduces factors other than those of pitch and the threshold value is not quite comparable to the threshold value in our method of constant stimuli. Professor Stumpf's curve, drawn by Meyer (5), shows exceptional ability and is probably accurate. The low thresholds can be adequately explained by the less extended range, by extraordinary natural capacity, and by a high degree of training in

experimental work. On the other hand it should be kept in mind that the curves of Stücker, Schaefer, and the writer, represent the results of a much larger number of observers, many of whom do not have exceptionally fine capacity for discrimination of pitch. The curves are therefore on a much higher level than if they were drawn exclusively from the results of observers who had unusually fine ability.

In the curves of Preyer, Schaefer, Meyer, and the writer the minimal threshold lies somewhere near the central region; but in the other two discrimination seems to be the best in the lowest level. With Preyer the finest capacity is at 500, with Luft at 64, with Meyer at 600 (although the thresholds for 200, 400, and 600 are practically equal), with Stücker at 73.4, with Schaefer at 128, and with the writer at 256 v.d. The maximal threshold is to be found in the highest part of the range in every case. The second maximum lies with Preyer, Meyer, Schaefer and the writer at 64 v.d.

An examination of these curves raises the question as to the cause of the variations. Individual differences are, of course, the principal cause but the nature of the objective control is undoubtedly a very important factor, especially at the extremes. If the apparatus and the method of the three investigators, who had a large number of observers, had been equally refined at the different steps, these grosser differences would probably not have occurred. As it is, they are most pronounced at the extremes where the control was the most difficult. The experimental control at 128, 256, and 512 v.d. can be made so perfect that no observer will be able to pass consistent judgments on any criterion other than pitch. For this reason the results of Stücker, Schaefer, and the writer agree, approximately, within this region.

Inasmuch as the curves take the same general direction, the variations in the upper limit are about what would be expected when one considers the difficulties to be encountered, together with the fact that one of the experimenters used an entirely different apparatus. But from 130.5 to 73.4 v.d., Stücker's curve continues in the same general direction which it has had throughout the entire course, while the other two curves have changed their direction. In other words, Stücker found the absolute difference for 73.4 v.d. to be less than for any other point in the line, while both Schaefer and I found at 64 v.d. the second maximum which is noticeably

greater than for any other point except at 2048 v.d. Luft's results seem to confirm those of Stücker, but Meyer's curve, as well as Preyer's, shows a rise at the lower limit. Indeed the ratio between the thresholds of Meyer for 100 and 200 v.d. is very similar to the ratio between the thresholds for 64 and 128 v.d. obtained by Schaefer and myself. I have no hesitancy in concluding, therefore, that sensitiveness in the great octave is, in general, not so keen as in the small octave. But for reasons already given, it does not follow that the difference is actually as great as the numerical results of this study would seem to indicate. In the light of the experience of the present study, however, Stücker's findings in the lower limit must be held in question. It seems more probable that his observers had learned to make judgments on some criterion other than pitch. Just what that may have been cannot be stated definitely as that author has failed to give any detailed account either in regard to method or to apparatus. It is only known that the tone in question was produced by a tuning fork. The possibilities of error with the large tuning forks are, however, sufficiently great to warrant the statement that Stücker's low record is due, not altogether to discrimination of pitch, but that secondary criteria have been operative in giving the low thresholds.

Individual Differences.—An examination of Table I discloses the fact that the observers may be classified in two general divisions. In the first there are thirty-seven whose curves follow the course of the composite curve in that the smallest values are to be found in the central register on either side of which a slow or a rapid rise is evident. In the second division, are thirteen whose curves do not conform to any general type. In the irregularity of the curves of this second division lies the only possible evidence of gaps which this study has developed.

The curves of the first division may, in a general way, be given a three-fold classification; namely, (1) those which show a relatively low threshold at some point in the central region and relatively high thresholds at the extremes, (2) those in which the thresholds are fairly uniform throughout the entire range, and (3) those curves in which the threshold for 64 is lower than for 2048 v.d.

Division I.—In Figure 4 are the five curves of the first group which show, the most strikingly, the relatively low thresholds in the central register and the higher thresholds at the extremes. These

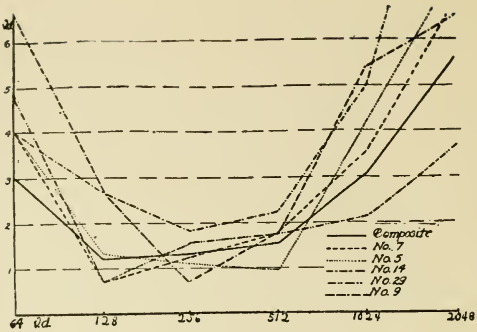

Fig. 4. Individual curves of observers Nos. 5, 7, 9, 14, and 27, which show relatively low values in the central region and high values at the extremes. The solid line is the composite curve of the fifty observers.

curves all resemble the composite more or less closely. At the extremes, however, all excepting that of No. 27 rise above the composite, but in the central region, at 128, 256, and 512 v.d., one-third of the fifteen thresholds pass beneath it. The normal variation of the point of keenest discrimination is well illustrated in this figure. Nos. 7 and 27 made the best record at 128, Nos. 9 and 14 at 256, and No. 5 at 512 v.d. In fact, all but one of the entire number of observers made their lowest record at one of these central levels.

These curves represent the results of observers who were the most unreliable. Very few of these values indicate the physiological threshold. One could not say that the high values at the extremes should be interpreted to mean that all of the observers in question were unable to perceive smaller differences on account of physiological incapacity. It is much more probable that the difficulty is psychological. Individuals of this type do not adapt themselves so readily to new situations under experimental control. When new adjustments must be made their work is relatively poor and continues on a low plane until time has been given for the proper adjustment after which their work may be on a par with that of individuals who adapt themselves more quickly to new situations.

Figure 5 represents the results of the six individuals who are most typical of the second group. All of these observers are men, but they are not of equal rank in previous work in discrimination of

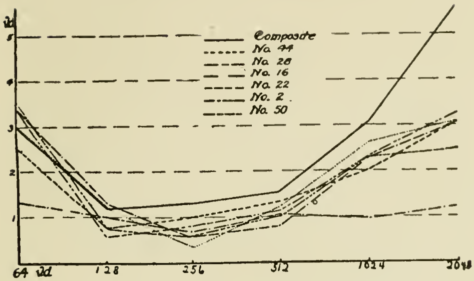

Fig. 5. The composite curve of the fifty observers and the individual curves of Nos. 2, 16, 22, 28, 44, and 50, which are characterized by a high degree of uniformity in the threshold values throughout the range.

pitch. Nos. 2, 32, and 48 were graduate students in psychology and were trained in other tests of discrimination; with the remaining three, however, previous training was very limited. The striking difference between these curves and those of the former group is, as would be expected, with reference to the extremes. The values for 64 and especially for 1024 and 2048 v.d. are lower than in curves of the first type; they approach, therefore, a more uniform level,—a goal which is most nearly approximated by No. 48. In contrast to the former group, these curves fall below the composite at practically every point; only four values are actually higher than the composite, while two more are equal, and these are at the lower limit. From the standpoint of consistency, the curves of Class II can easily be judged the better. Observers who give such results are reliable. With a state of secondary passive attention, they are able to meet the new situation in an easy and natural manner and are little disturbed by unusual difficulties which may be presented. In addition, exceptional ability in analyzing a problem enables them to select the proper element or elements upon which to base their judgments, even though there be disturbing factors. They are so consistent that the experimenter can feel a high degree of assurance that their records represent a close approach to the physiological threshold.

The curves of the five individuals who are most representative of the third group are shown in Figure 6. The peculiar character of these curves, in contrast to those already considered, lies in the lower limit. Here the thresholds are very much lower than for Class I

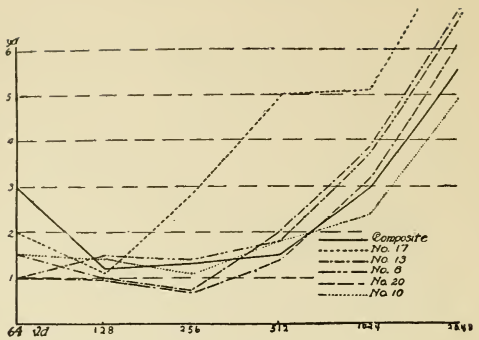

Fig. 6. The composite curve of the fifty observers and the individual curves of 8, 10, 13, 17, and 20, which show relatively low values at the lower limit and high values at the upper limit.

and considerably lower than they are for Class II. But in the upper extreme the curves are similar to those of the first group, with one exception,—the curve of No. 17. Indeed the average results of these five observers form a curve which closely approximates Stücker's curve, the essential difference being, that the latter is tilted at a slightly different angle, due to the fact that Stücker's thresholds at 73.4 v.d. are lower than ours and higher in the vicinity of 2048 v.d.

The similarity of our results to those of Stücker in the lower extreme might invite the same criticism which we advanced against him. It might be said that our low threshold at 64 v.d. was due to the discovery of some variable other than pitch upon which the judgment was based. There is, of course, the possibility that this occurred, but reference to Figure 7, in which the composite curves of the three groups may be compared, leads to the belief that such a criticism does not have much weight with respect to these particular observers. It is to be observed that the minimal thresholds of the first two groups lie at 256 v.d. with a gradual rise on either side of this point. The point of keenest discrimination for Class III, however, lies at 128 v.d. with here again a rise on either side proportional to that which we find in the other two groups. In other words, the form of the latter curve from 64 to 256 v.d. is similar to the form of the other two from 128 to 512 v.d. We should expect to find a higher threshold for 64 when the minimum is

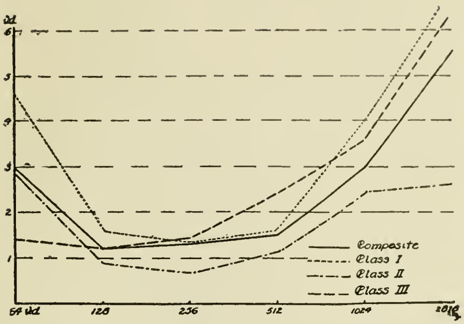

Fig. 7. The composite curves of the fifty observers and the three different classes.

at 256 than when it is at 128 v.d., and this is what occurs. With conditions such as they are, we are inclined to regard the results of this class of observers, at 64 v.d., as fairly accurate. The observers in question naturally do better work on the low tones. If such an interpretation is true, it would not be just to say that the curve of Class III is less consistent than the curve of Class II; each represents a different type and is consistent with itself throughout.

There are still to be considered the results of the thirteen observers which are not exactly comparable to any of the classes described above, because of certain irregularities occurring in their curves. It must be determined whether or not these ridges or elevations may be explained on the basis of daily fluctuations, in which case a sufficiently large number of observations would result in smooth curves, or whether the variations are due to natural weaknesses for the regions where they are found. The extent to which this latter explanation must be invoked, indicates an answer to the question of the frequency of gaps in the registers of individuals who have apparently normal hearing.

In Table I, the observers who have such curves are Nos. 4, 8, 11, 12, 18, 19, 25, 38, 40, 41, and 45. It will be noted, however, that at no place are the deviations from the normal very pronounced. All of them, excepting perhaps one, are doubtless due to certain factors which would have been eliminated by a large number of judgments. Daily variations, the relative amount of practice, and the accuracy with which the increments used corresponded to the true

thresholds, are the most important factors which have contributed toward the irregularities. With No. 5, for example, the experiment was begun at 256 v.d., so that the greater amount of practice would give the neighboring tones the advantage. With No. 11, the tone of 256 v.d. was first tried with a difference of 2 v.d., which was too large, resulting in an almost perfect series. Had the order in which these differences were presented been reversed, the threshold would probably have been very close to 1 v.d. Indeed, a number of these observers were given additional tests to determine whether or not these variations from the normal would hold. In each case as Table III will show, the curves became fairly smooth.

The curve of No. 4 is abnormal at 1024 v.d. in its relatively low threshold of 1.1 v.d. During an experimentation of one hour he made a record of eighty-eight per cent. on two hundred judgments with a difference of 2 v.d. But on the following day, a difference of 1 v.d. gave only fifty-two per cent. of the right cases. A larger number of observations would doubtless have resulted in a threshold more equal to that of the tone an octave lower.

But there is one observer, No. 18, who gives some evidence of a

TABLE III. Irregular results which additional observations have corrected

Observer	*64*	*128*	*256*	*512*	*1024*	*2048*
19	5.2	0.6	1.1	0.8	4.1	9.7
		0.6	0.6	1.1		
41	2.4	0.8	0.7	2.1	1.8	3.5
				1.8	3.5	
35	6.4	1.1	1.5	1.1	6.4	9.9
		1.8	2.2	2.6		
38	0.7	0.6	0.7	2.1	1.6	3.7
			1.1	.9	1.3	
12	3.7	0.7	1.5	2.4	4.3	3.5
				1.8	3.3	

slight weakness in the region of 256 v.d. His threshold at this point was derived from four hundred judgments. The first half of the number with a difference of 3 v.d. gave a threshold of 2.9 v.d., while the second half with a difference of 2 v.d. gave a threshold of 2.3 v.d. At no time was he able to approach a threshold of 1 v.d. On the other hand, with the tones above and below, he made low and consistent thresholds. It is difficult to account for this high threshold at 256 v.d.; the observer himself could offer nothing as a basis for explanation. The affective element, association and imagery, and inherent characteristics of volume and intensity may have played varying rôles in causing the discrepancy. At any rate the differ-

ences are not sufficiently great to be regarded as representing gaps.

We have found, then, from this study of the curves of discrimination of pitch of fifty normal observers no clear evidence of tonal gaps. The grosser irregularities which might arouse the suspicion of a gap are due to certain factors which have not been perfectly controlled. It is highly probable that with more extended observations the irregularities would have been eliminated. It must be kept in mind, however, that this conclusion has reference only to observers with apparently normal auditory capacity; with respect to individuals whose audition is unquestionably recognized as pathological, this study has nothing to offer.

Relation of Musical Training and Expression to Discrimination of Pitch.—The question naturally occurs in a study of this kind as to the nature and extent of the correlation between musical education and pitch-discrimination. It seemed obvious that if a correlation existed it would be between discrimination and musical expression rather than between discrimination and mere technical training. The Pearson method of rank difference was used to determine the correlation. The mean of the six levels in the range for each of thirty-eight observers gave a value for the ranking of the individuals according to their capacity for the discrimination of pitch. The records of the remaining twelve were not included as most of them were advanced students whose greater experience in work in the laboratory might possibly put them in a slightly better class, while with one or two others, information regarding their musical training was not at the time available. The ranking according to expression was not quite so simple. For this purpose an evaluation was made of the answers to the questionnaire, which was an exact duplicate of the one published by Professor Seashore in his Preliminary Report (13). To recall, there are three questions under the topic "Musical Expression": namely, (1) Favorite selections you can sing (by ear? by note?), (2) Favorite selections you can play (by ear? by note?), (3) Singing or playing in public (parts, occasions, etc.). The individuals were instructed to give as specific information as possible. But the comparison of the two functions showed no correlation whatever.

It was still believed, however, that there must be some difference between the discriminating capacity of those who seemed to be the

most musical and those who appeared to be the least, as far as previous experience was concerned. Again the questionnaire, to which reference has been made was resorted to, but this time the questions were designed to reveal the amount of training. They were as follows: (1) Musical training in public schools, (2) Private vocal lessons (when, where, how long, etc.), (3) Private instrumental lessons (when, where, how long, etc.). The observers were then equally grouped in two divisions, the first group consisting of the better ones in training and expression and the second of the poorer ones. The mean threshold for each group for the different levels is recorded below:

Table IV

V.d.	64	126	256	512	1024	2048
Group I	3.8	1.2	1.2	1.8	3.8	6.8
Group II	3.1	1.5	1.5	1.6	3.1	4.9

We find, then, that the group whose members have had greatei musical education and more practice excel in capacity for discriminat-

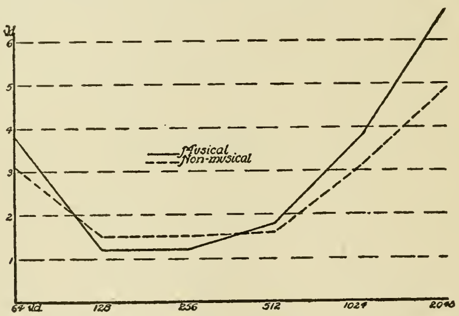

Fig. 8. Comparison of the musical and the non-musical.

ing pitch only at 128 and 256 v.d. But it is in this region that the above factors would have the most influence. Their effect upon the differential threshold of either extreme would be small because these tones are seldom used either in singing or playing. There is, then, some correlation between musical ability and discrimination of pitch in the central register. This is in general agreement with the conclusions of both Mount (6) and Smith (15) who found a fair degree of correlation between musical expression of pitch and dis-

crimination of pitch at 435 v.d. But we cannot agree with Stücker (17) in his assertion that musical observers, in general, show keener discrimination in the upper limit than the non-musical ones. Much depends upon the standard of classification for musical observers. Stücker may have reference only to observers of unusually fine ability in music. For such, his statement may be true, but for observers whose ability is not so exceptional it scarcely holds.

The frequency and distribution of the false judgments.—From the 39700 judgments it has been possible to determine definitely not only the frequency of the false judgments but also the way in which they have been distributed in the various levels. So large an amount of data should show whether or not there is a preference for one or the other order, and if so what relation this preference bears to sex, voice-register, and pitch.

In computing the number of errors, the results of sixty-two individuals, thirty-two women and thirty men, for at least three different tonal regions were available. The nature of the error in the wrong judgments at 64 v.d. was not recorded; the observer sat with closed eyes and gave oral judgments and the experimenter merely recorded the number of the errors. At the other levels, with one or two exceptions, the observer recorded *H* or *L* for each pair of tones. For 128, 256, 512, 1024, and 2048 v.d., then, the distribution of errors could be accurately studied. Just one-half of the sixty-two observers had a record for each of these five steps, for the other half discriminations were made at from three to four levels. Two different computations were therefore made, the first including the results of the entire number of observers and the second, only those which are complete for the five different levels. The total of the complete results could thus be used as a check upon the total of the incomplete results. Reference shall be made to the first, however, only in so far as it differs from the second.

TABLE V. Distribution of errors

Section 1

Computed from the results of sixty-two observers

A	B	C	D	E
128	8300	23.40	10.78	12.62
256	8600	25.13	13.31	11.82
512	10100	26.03	13.79	12.24
1024	8300	25.42	13.61	11.81
2048	4400	25.41	14.00	11.41
Total	39700	25.09	13.04	12.05

Section 2

Computed from the results of thirty men

128	4700	23.21	10.34	12.87
256	4800	25.00	12.83	12.17
512	6100	27.46	13.84	13.62
1024	4500	24.67	12.50	12.17
2048	2100	23.52	11.61	11.91
Total	22200	25.11	12.39	12.72

Section 3

Computed from the results of thirty-two women

128	3600	23.67	11.36	12.11
256	3800	25.29	13.92	11.37
512	4000	23.78	13.48	10.30
1024	3800	26.32	14.95	11.37
2048	2300	27.13	16.13	11.00
Total	17500	25.07	13.86	11.21

Table V is a record of the errors computed from the results of the total number of the observers. Column A represents the vibration-rate of the fork; B, the total number of judgments; C, the total percentage of error; D, the percentage of error when the second tone was lower; and E, the percentage of error when the second tone was higher. The greater number of judgments in the central register is due to the fact that irregularities occurring here necessitated further experimentation to determine whether they were due to subjective factors which were permanent or merely transient, or possibly to objective factors.

The final average of the percentage of right cases approaches to within .09 per cent. of the ideal of 75 per cent. When the different levels are considered collectively, the false judgments amount to 13.04 per cent. when the second tone is lower, and to 12.05 per cent. when the order of succession is reversed. There seems then to be a slight though not significant preference for the order in which the second tone is higher.

A difference is observable in the distribution of error at the various levels. At 128 v.d. more errors by 1.84 per cent. occur when the second tone is higher, but at the other levels there is a greater percentage of error with the opposite order. As shown in Table V the differences between Column D and E increase gradually from 256 to 2048 v.d. In the first computation, however, made from the complete results of a smaller number of observers, the order of second tone higher gives the smaller per cent. of error at each level. At 128 v.d., the difference in favor of this order is only .48 per cent., but

at 256 it amounts to 2.03 per cent., at 512. to 3.65 per cent., at 1024 to 2.68 per cent., and at 2048 v.d. to 2.46 per cent. On the average, then, judgments of difference in pitch are more accurate when the second tone is higher, *i.e.* given two successive tones of the same pitch, there is a slight tendency to hear the second as the higher, excepting at 128 v.d., where fewer errors are made when the reverse order is followed.

Difference of sex.—When the results are studied with respect to sex it is found that the above conclusion would not be valid for a group of individuals in which there was a much larger percentage of men than women.

In the study of differences of sex it is found that the women on the average, show a decided preference for the order in which the second tone is higher at every step except at 128 v.d., where the difference seems to be slightly in favor of the second tone lower. But it is at this latter level that the men show a very strong preference for the second tone lower, while in the other levels the difference in favor of either order is insignificant. This variation of sex affords additional evidence that normal illusions are greater with women than with men.

An arrangement of results according to voice registers of the observers brought out nothing new. The difference seems to be essentially between the voices of men and women. Had our observers been highly specialized singers, there might have been some difference showing itself in the different voice registers.

TABLE VI. Variation with sex

V.d.	*64*	*128*	*256*	*512*	*1024*	*2048*
20 women	3.2	1.2	1.4	1.7	3.8	6.6
16 men	2.7	1.1	1.0	1.6	2.4	4.8

The foregoing table and the accompanying figure show the results for the twenty women and the sixteen men who had a similar amount of training in experimental procedure. At first sight there seems to be a decided difference between the sexes, inasmuch as the thresholds for men are lower throughout the whole range than those for women. While there are differences in favor of the men, care must be taken not to attach too much significance to them. The differences at 1024 v.d., of 1.4 and at 2048 v.d. of 1.8, seem to be considerable, yet they are not much greater than should be expected when the total results are considered. At 256 v.d. the varia-

tion of .4 appears high when compared with the difference of .1 in the octave just above and just below. With such noticeable variation in the central region it is not so surprising to find much larger differences at the extremes where objective factors are not so well controlled. Smith (15) reports practically the same difference of sex as is shown in these results. He finds that at the ages of 17 to 20 and at maturity, the men surpass the women by an average of 0.3 v.d. at 435 v.d. It is evident that the men's curve presents a more satisfactory form than does that of the women, in that there is not so high a variation between the points of keenest discrimination and the

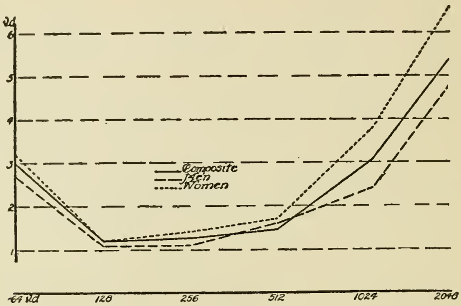

Fig. 9. The comparative curves of twenty women and sixteen men together with the composite of the fifty observers. (Table VI).

extremes. It may be that the cause of this arises from a possible inherent difference between the sexes in the method of meeting new situations. Or it may be that the men adapt themselves more quickly to experimental conditions and for this reason it has been easier to reach their physiological threshold.

Stücker (17) contends that the greatest sensitiveness to small differences of pitch lies with tenors and sopranos in the lower half of their voice registers, but with singers of bass and of alto parts, as a rule, in the upper half. In other words, the differences is not between the voices of men and the voices of women but between the relative height and depth of the voice register of both sexes. As none of my observers could be classed as professional singers, the results have little to offer either positively or negatively, in regard to Stücker's statement. Table VII indicates that only the results of

the soprano singers can be harmonized with the conclusion of Stücker. The finest sensitivity of the tenors is in the central part of their register and not in the lower, as he finds it to be; the basses made the best record in the lower part of their register, rather than in the upper; the baritones have done better in their register; and finally, the altos do better in the lower register and not in the upper. But these facts are not necessarily contradictory to Stücker's, inasmuch as the observers in this experiment represent only average ability as singers. One should have plotted the curves of a relatively large number of highly practiced singers before he would be able to add a conclusive word in answer to the problem which Stücker has suggested.

TABLE VII. Average thresholds classified according to voice register

Soprano	(16)	3.8	1.5	1.6	2.1	4.1	7.1
Tenor	(4)	4.1	1.5	1.2	1.2	2.3	4.3
Baritone	(15)	3.2	1.4	1.3	2.1	2.4	4.7
Alto	(8)	3.5	1.5	1.2	1.3	4.3	6.0
Bass	(7)	2.5	1.1	1.3	1.6	2.8	4.3

SUMMARY

(1) For individuals selected because of a slight superiority at 435 v.d., the composite absolute curve of pitch-discrimination within the limits of 64 and 2048 v.d. shows the keenest discrimination at 128 and 256 v.d. On either side of this central register, there is a rise in the curve which is relatively abrupt toward the lower limit but much more gradual toward the higher extreme.

(2) The relative curve takes the form of a continual decline from the lower to the higher limit. From 64 to 128 v.d. the decline is comparatively steep, but from 128 to 2048 v.d., it is very gradual, approaching approximately a horizontal line in the upper half of the register.

(3) Individual differences, factors which lead to confusion and to identification, and variation in practice and in attention are the principal conditions upon which the form of the curve depends. The variations in the curves of the different investigators are explainable on the basis of the varying degrees of influence of these conditions.

(4) Most of the individual curves conform more or less closely to one of the following types of curves; namely, (a) a curve in

which there is a relatively low value at some point in the central register and relatively high values at the extremes, (b) a curve in which the thresholds are fairly uniform throughout the entire range, and (c) one in which the threshold for 64 is considerably less than for 2048 v.d.

(5) There is very little evidence of tonal gaps. The grosser irregularities in a few curves, which at first seemed to indicate the presence of a gap, disappeared with more extended observations.

(6) A correlation between musical ability and discrimination of pitch occurs only in the central register.

(7) The women make more accurate judgments when the second tone is higher; their preference for this order increases in direct proportion to the pitch, within limits, excepting at 128 v.d. where the reverse order is slightly preferred. The men make fewer mistakes at 128 v.d. when the second tone is lower, but at the other levels no particular preference for either order of succession is observable.

(8) The men surpass the women in discrimination of pitch at every level in the register; this variation between the sexes is the greatest at the extremes.

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THE DURATION OF TONES, THE TIME INTERVAL, THE DIRECTION OF SOUND, DARKNESS AND QUIET, AND THE ORDER OF STIMULI IN PITCH DISCRIMINATION

BY

DAVID ALLEN ANDERSON

I. Most favorable duration of the tones

In this investigation to ascertain the relative favorableness of different durations of tone in pitch discrimination, the tones were produced by tuning forks from the "standard pitch discrimination set" as described by Professor Seashore (1) reenforced by Koenig adjustable resonators suspended behind a revolving slit-disc which was driven by a synchronous motor (2,3).

The tuning forks were tuned to an accuracy of $\pm$ .015 v.d. They were held firmly by the fingers near the end of the stem and energized by striking the middle of the prong lightly against a sounder made of ¾ in. lead pipe covered with a soft rubber tubing and resting on a leather cushion filled with sand. When they had been set in motion the forks were held directly in front of the mouths of the resonators during the passage of the open slits in the intervening revolving disc. Revolving discs made from cardboard, in which were slits cut in appropriate sectors, regulated the duration of tones and the interval between them. The disc proper prevented the passage of the vibrations from the forks to the resonators while the slits admitted of their free passage. The length of the slit determined the duration of the tone and the size of the sector between governed the length of the interval. When a slit passed a fork the resonator would take up the vibrations. The result was a clear and pure tone, clean cut at beginning and end. The intensity was kept as regular as possible without maintaining an identifiable uniformity. An effort was made to change the forks from hand to hand and to govern the duration of time between the energizing of the forks and the hearing of the tones in such a way that the observers could get no clue regarding the order in which the tones were to be given. Whether the higher or lower tone was to be given last was regulated by a key prepared beforehand according to chance, except that not more than three consecutive cases of one kind were allowed.